Pointwise Assouad Dimension

~ and regularity of invariant sets ~

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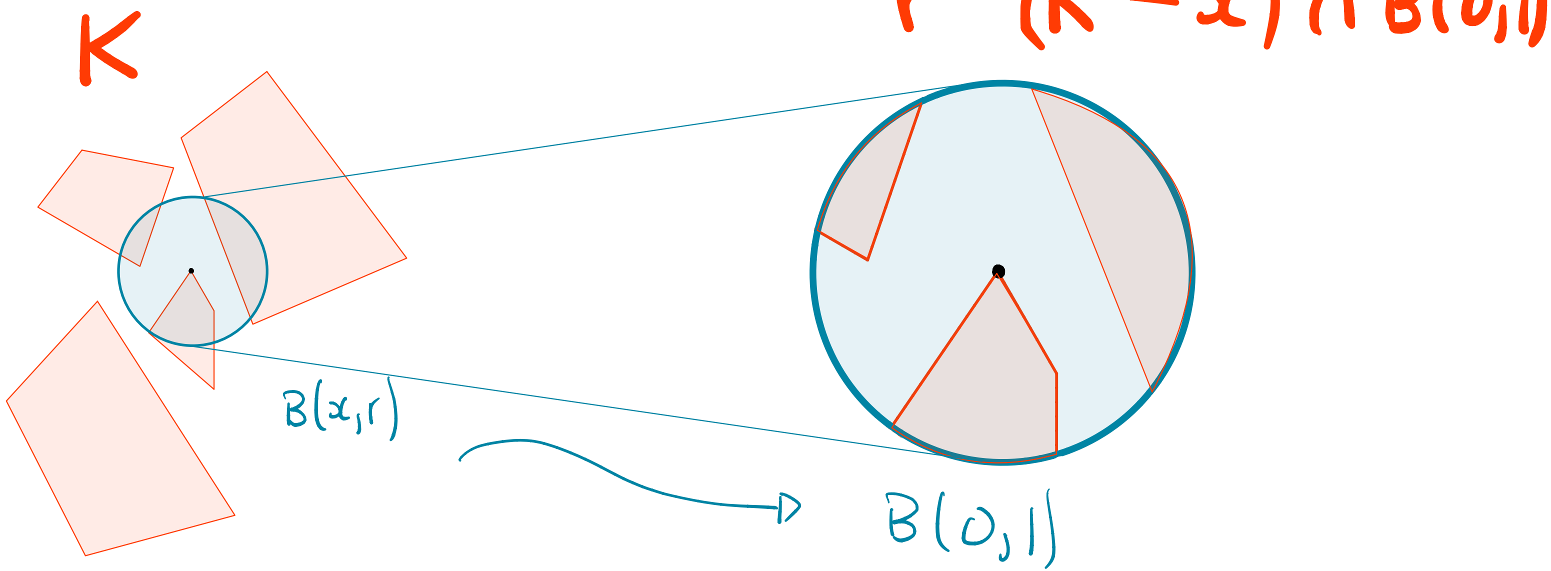
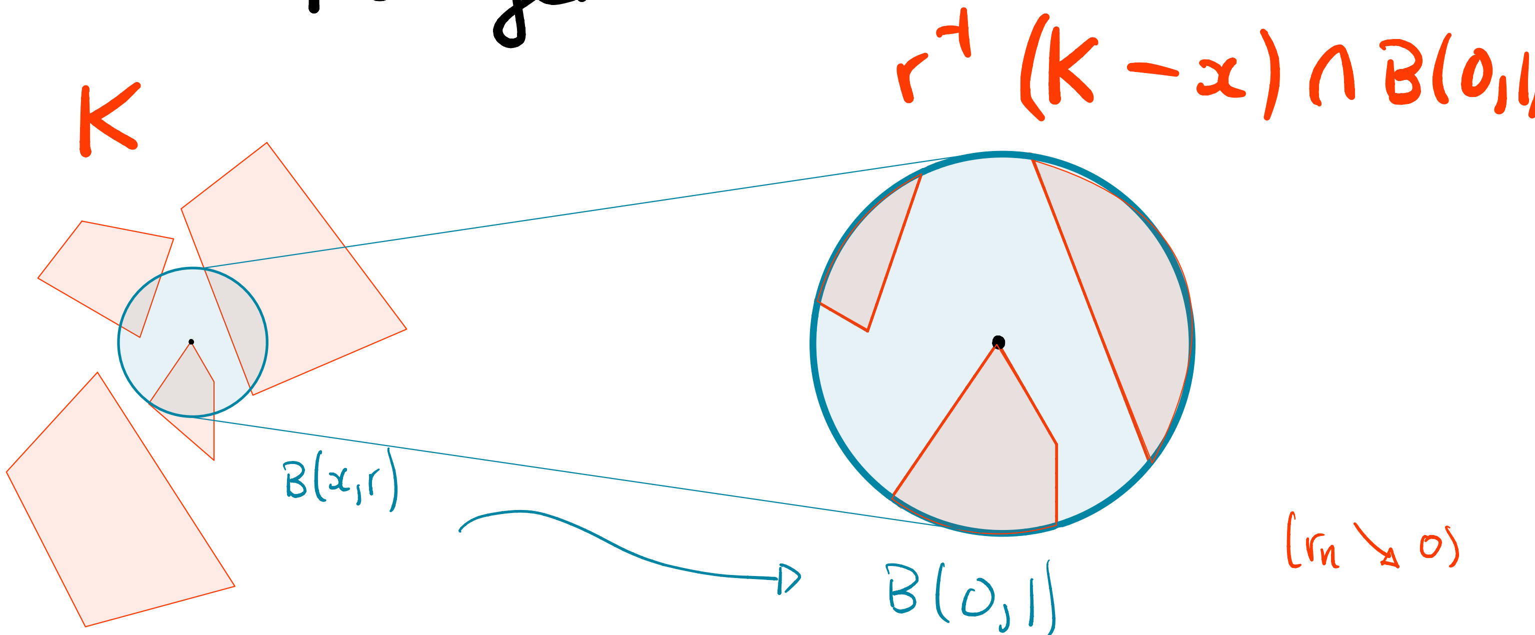
(Weak) Tangents



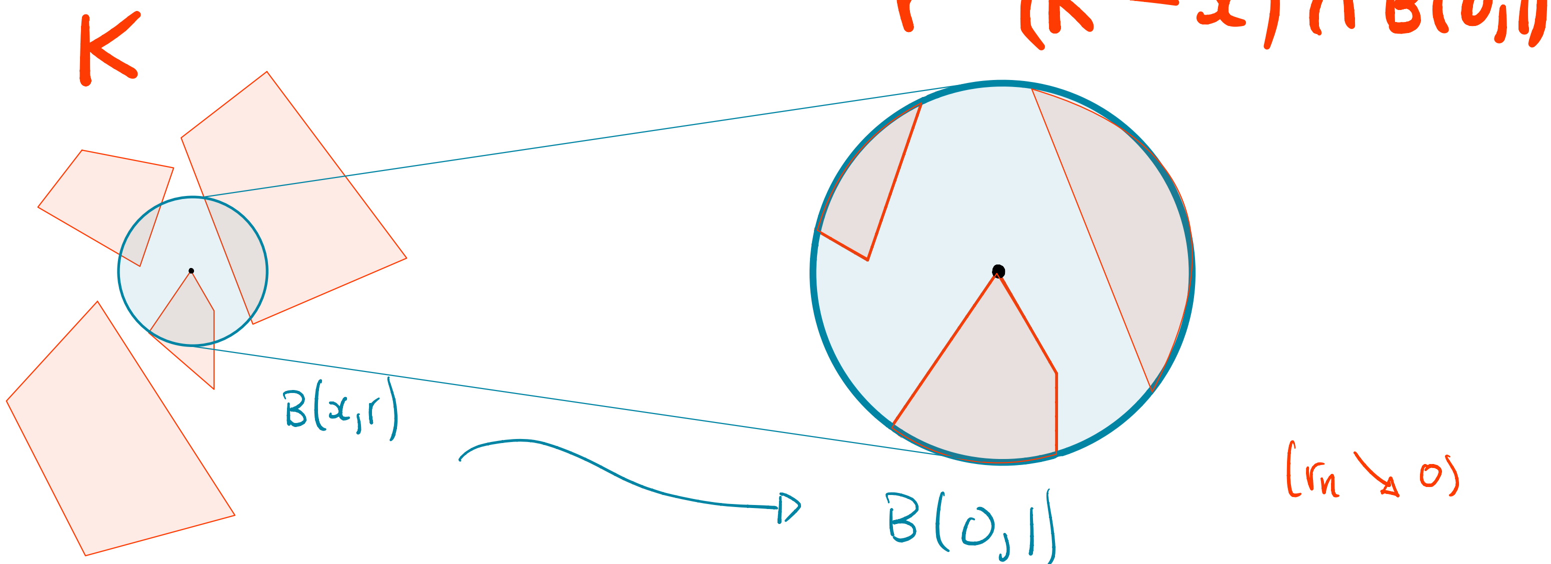
Tangents



Tangent: $\lim_{n\to\infty} r_n^{-1}(K-x)\cap B(0,1)$

(in Hausdorff distance)

Weak Tangents



Weak Tangent: $\lim_{n\to\infty} r_n^{-1}(K-x_n)\cap B(0,1)$

(in Hausdorff distance)

$$\dim_A K = \inf\left\{ \alpha : \forall\, 0 < r \leq R < 1 \ \forall x \in K \quad N_r\big(B(x,R) \cap K\big) \lesssim \left(\frac{R}{r}\right)^{\alpha} \right\}$$

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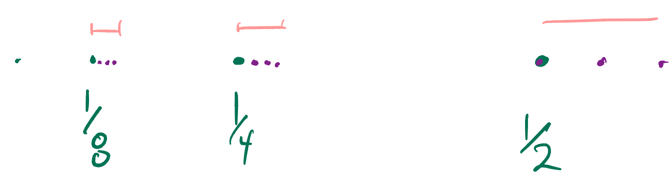
NOT NECESSARILY FOR TANGENTS!

$\delta_n \to 0$ faster than $\frac{1}{2^n}$

e.g



Self-embeddable: $\forall B(x,r)$, $x \in K$

$\exists$ bi-Lipschitz $f: K \longrightarrow B(x,r) \cap K$.

e.g. Any attractor of contracting bi-Lipschitz IFS.

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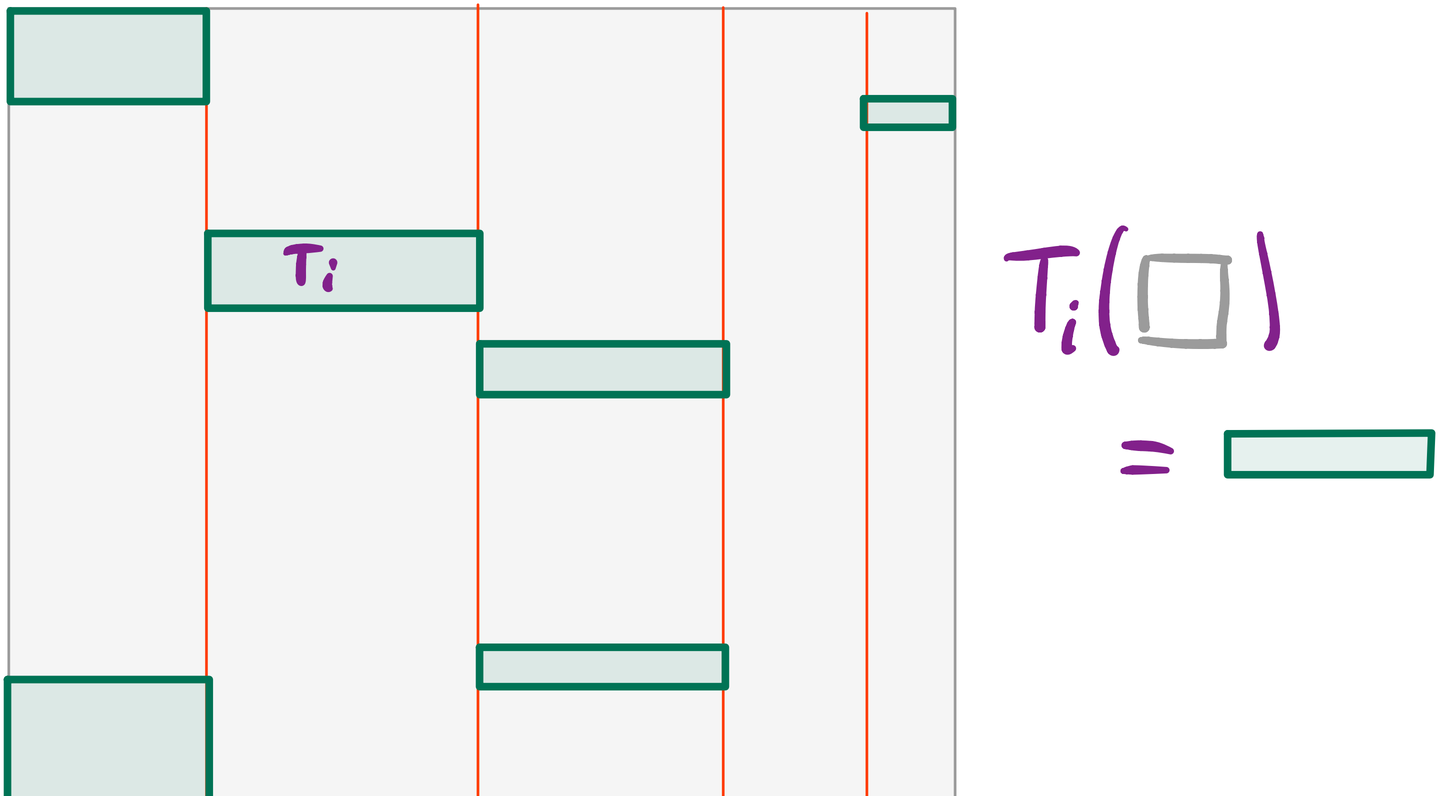
Käenmäki + R. (2023+): Suppose K is self-embeddable. Then:

$$\dim_A K = \sup\{\dim_H F : F \text{ tangent of } K\}$$

Example: K Gatzouras–Lalley carpet



Example: K Gatzouras–Lalley carpet

$$\mathcal{H}^{\dim_H K}\left(\left\{x \in K : \sup\{\dim_H F : F \in \mathrm{Tan}(K,x)\} < \dim_A K\right\}\right) = 0$$

BUT

$$\dim_H \left\{x \in K : \sup\{\dim_H F : F \in \mathrm{Tan}(K,x)\} = \alpha\right\} = \dim_H K$$

(for $\dim_H K \leqslant \alpha \leqslant \dim_A K$) (non-empty $\Longleftrightarrow$ $\dim_L K \leqslant \alpha \leqslant \dim_A K$)