Assouad Spectrum of Gatzouras–Lalley Carpets

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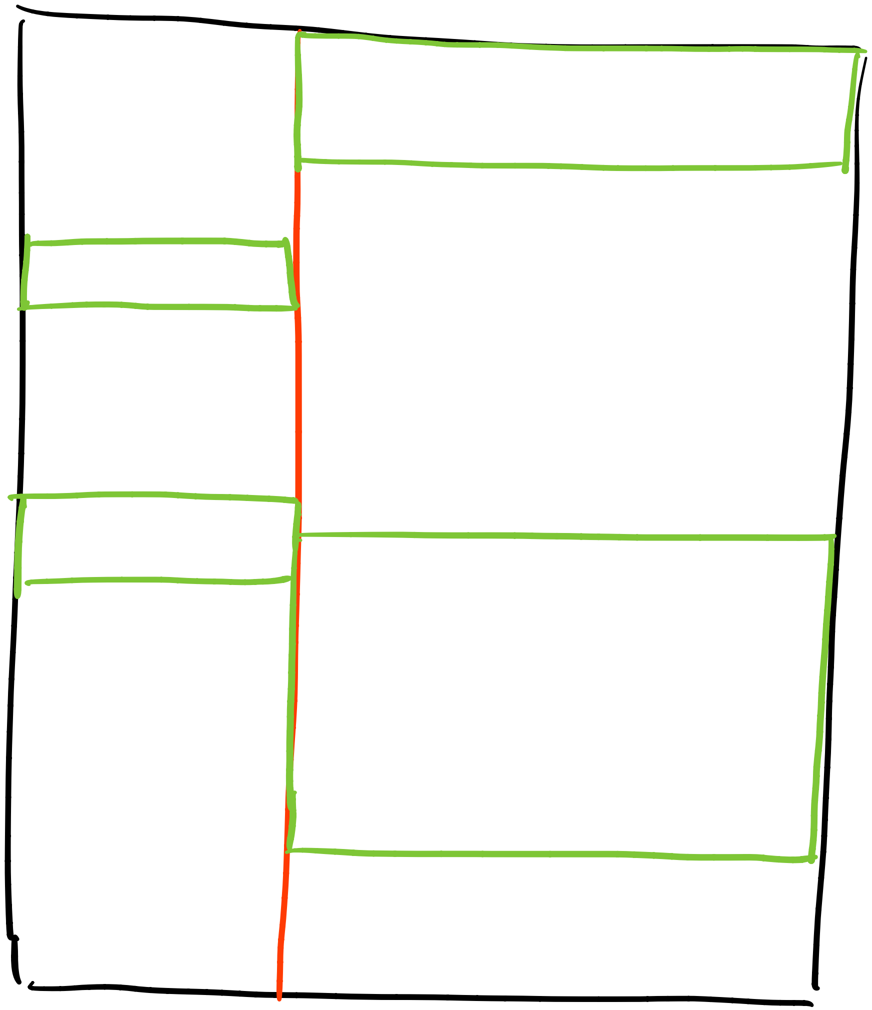
Fix $\theta \in (0,1)$

Assouad Spectrum:

"Box-dim scaling" from $r^\theta \to r$.

$$\dim_A^\theta K = \limsup_{r \to 0} \frac{\log \sup_{x \in K} N_r\left(K \cap B(x, r^\theta)\right)}{(1-\theta)\log(1/r)}$$

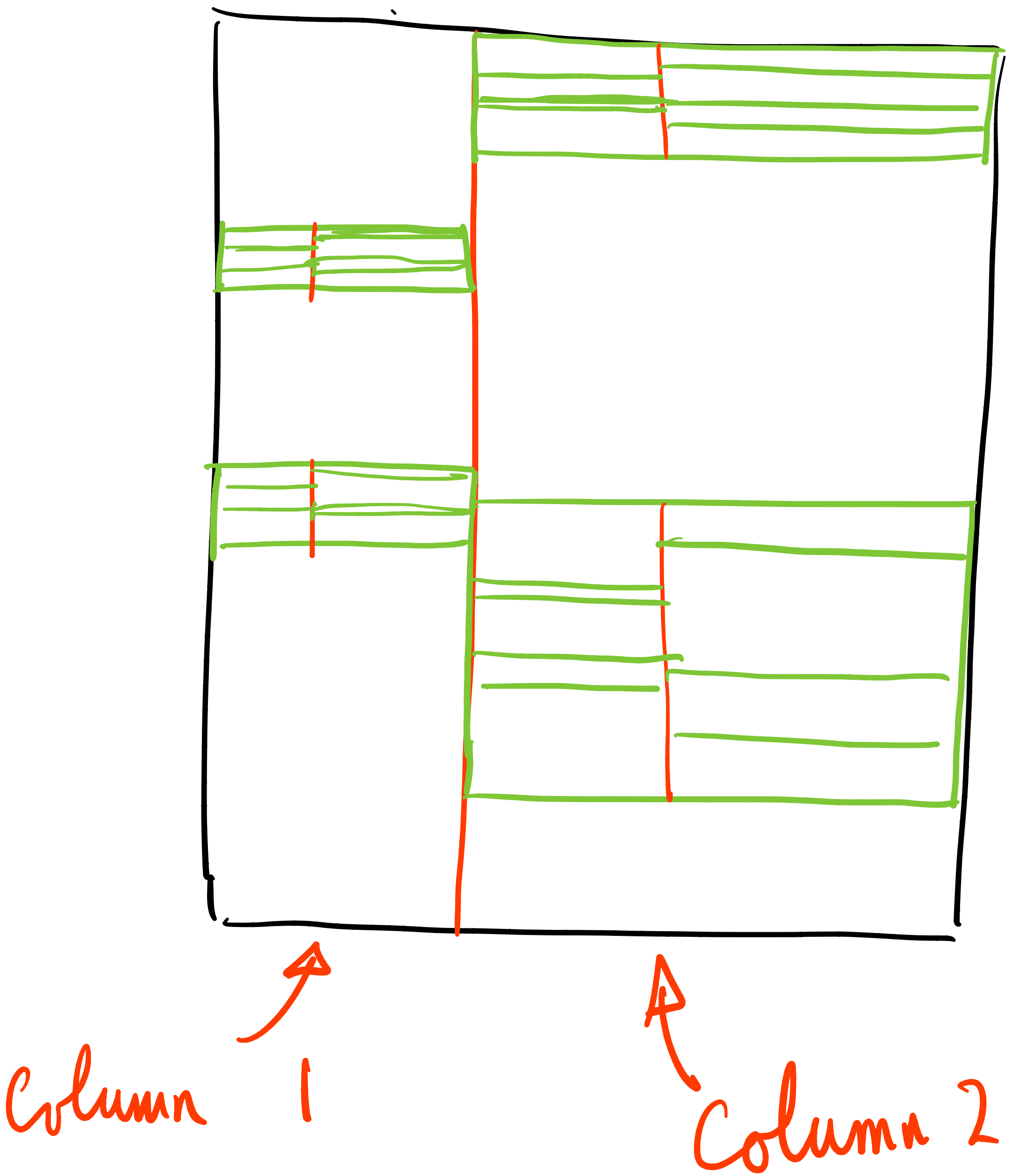
Gatzouras — Lalley Carpet



$$T_i : \square \to \boxed{}$$

$$K = \bigcup_{i \in \mathcal{I}} T_i(K)$$

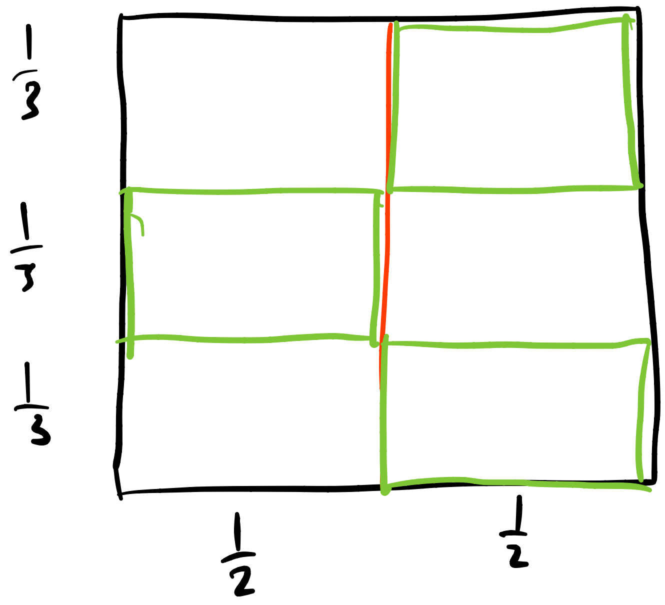
Gatzouras – Lalley Carpet



$T_i : \square \longrightarrow ▭$

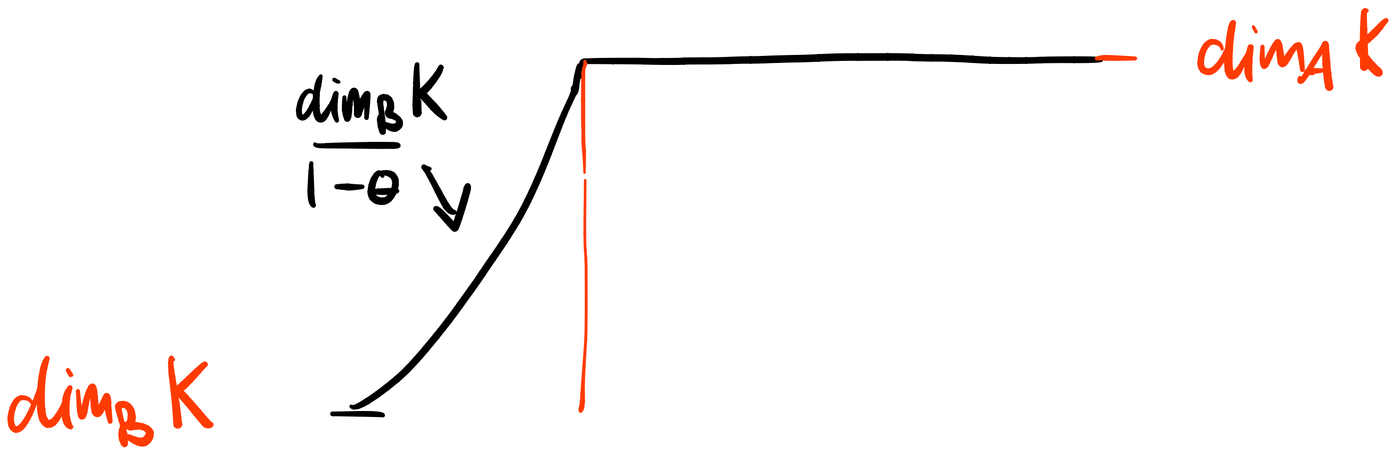
$$K = \bigcup_{i \in \mathcal{I}} T_i(K)$$

Special Case:



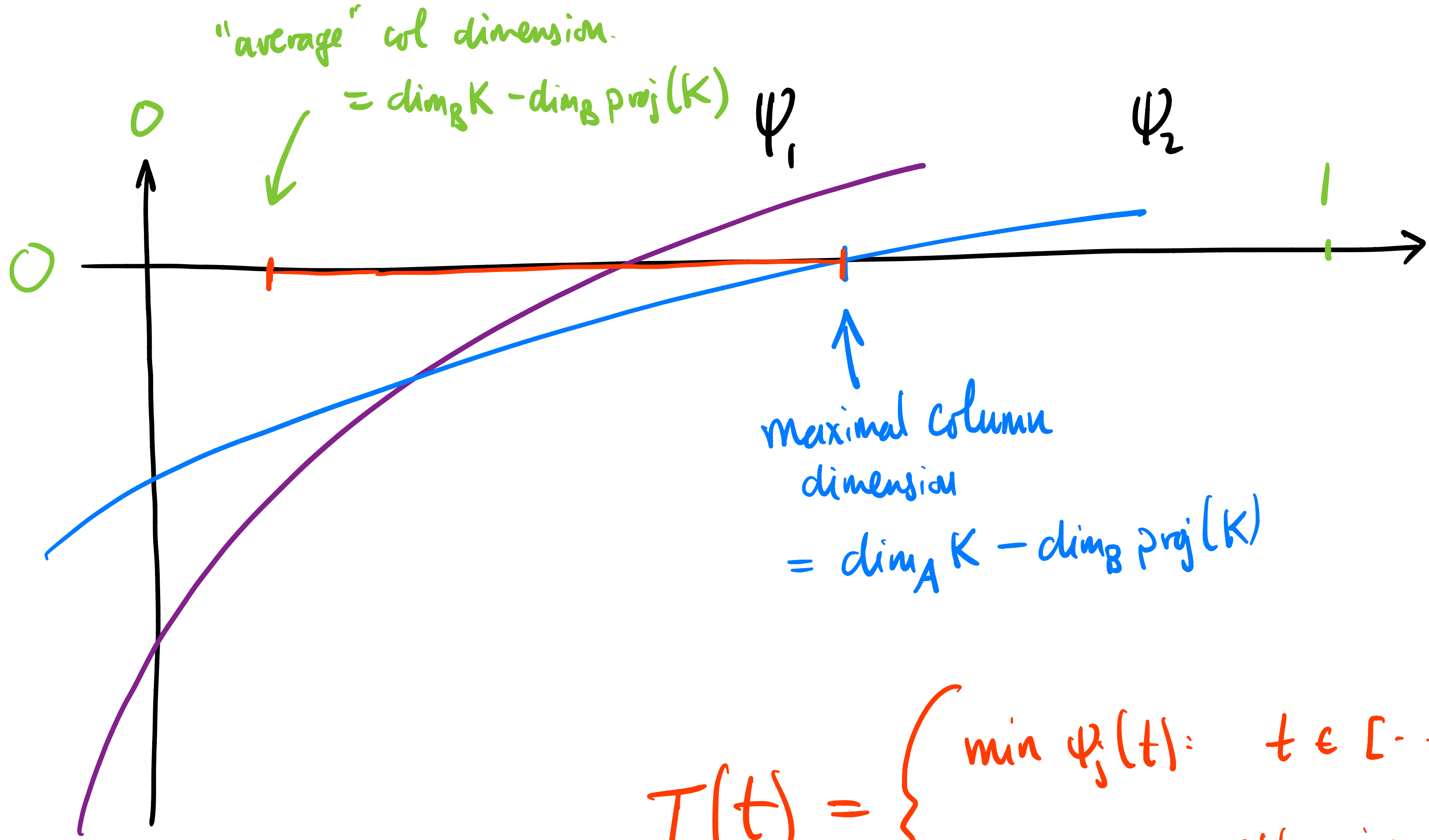
Proposition: Fraser, Yu 2018.

width(i)
height(i)
constant.



Column Pressure

Column j: $\psi_j(t) = \dfrac{\log \sum_i \text{height}(i)^t}{\log \text{width}(j)}$



$$T(t) = \begin{cases} \min \psi_j(t) : & t \in [\cdots] \\ -\infty : & \text{otherwise} \end{cases}$$

Theorem [Banaji–Fraser–Kolossváry–R. 2024+]

$$\dim_A^\theta K = \dim_B \operatorname{proj}(K) + \frac{T^*(\phi(\theta))}{\phi(\theta)}$$

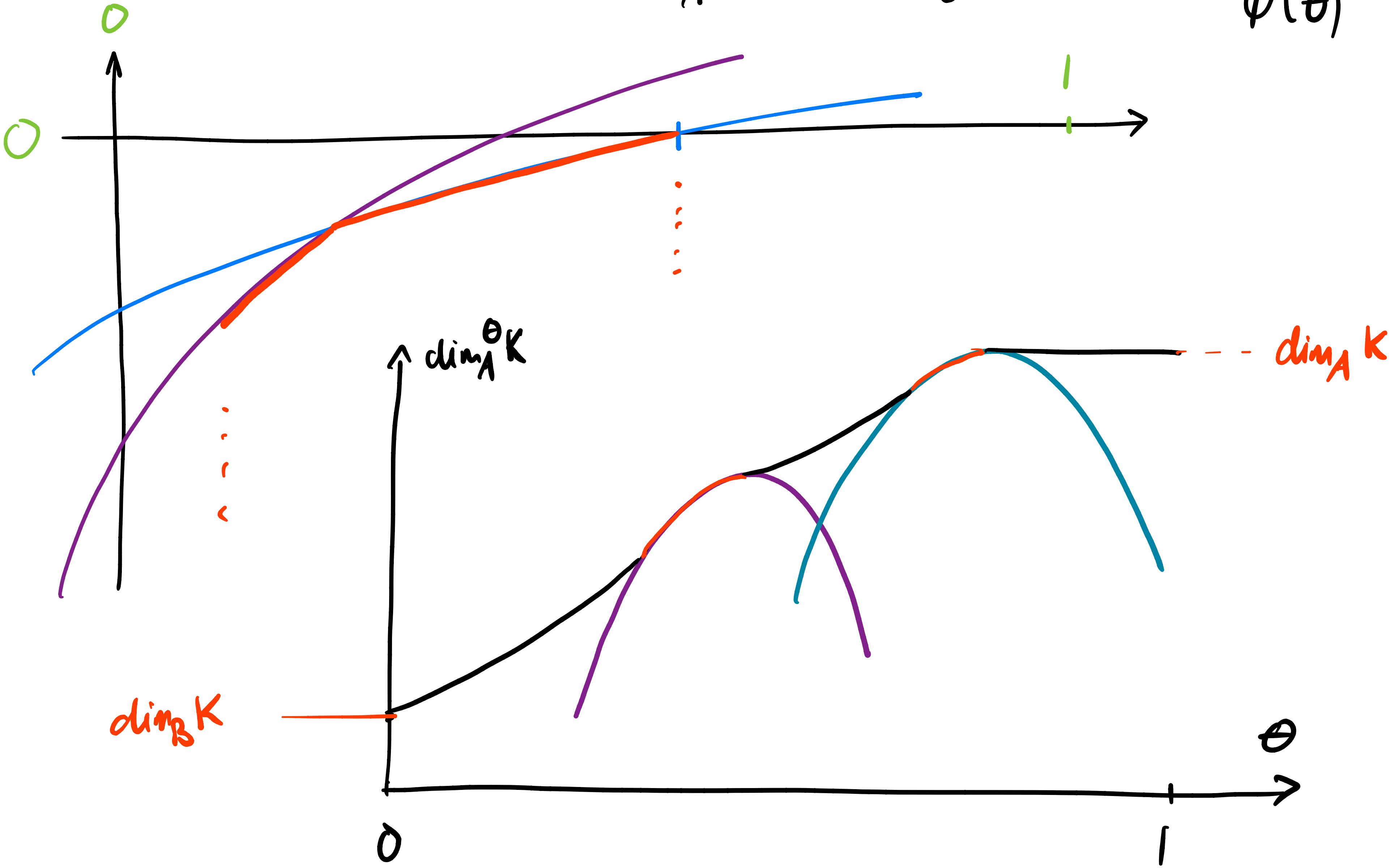
where $\quad T^*(\alpha) =$ Concave Conjugate

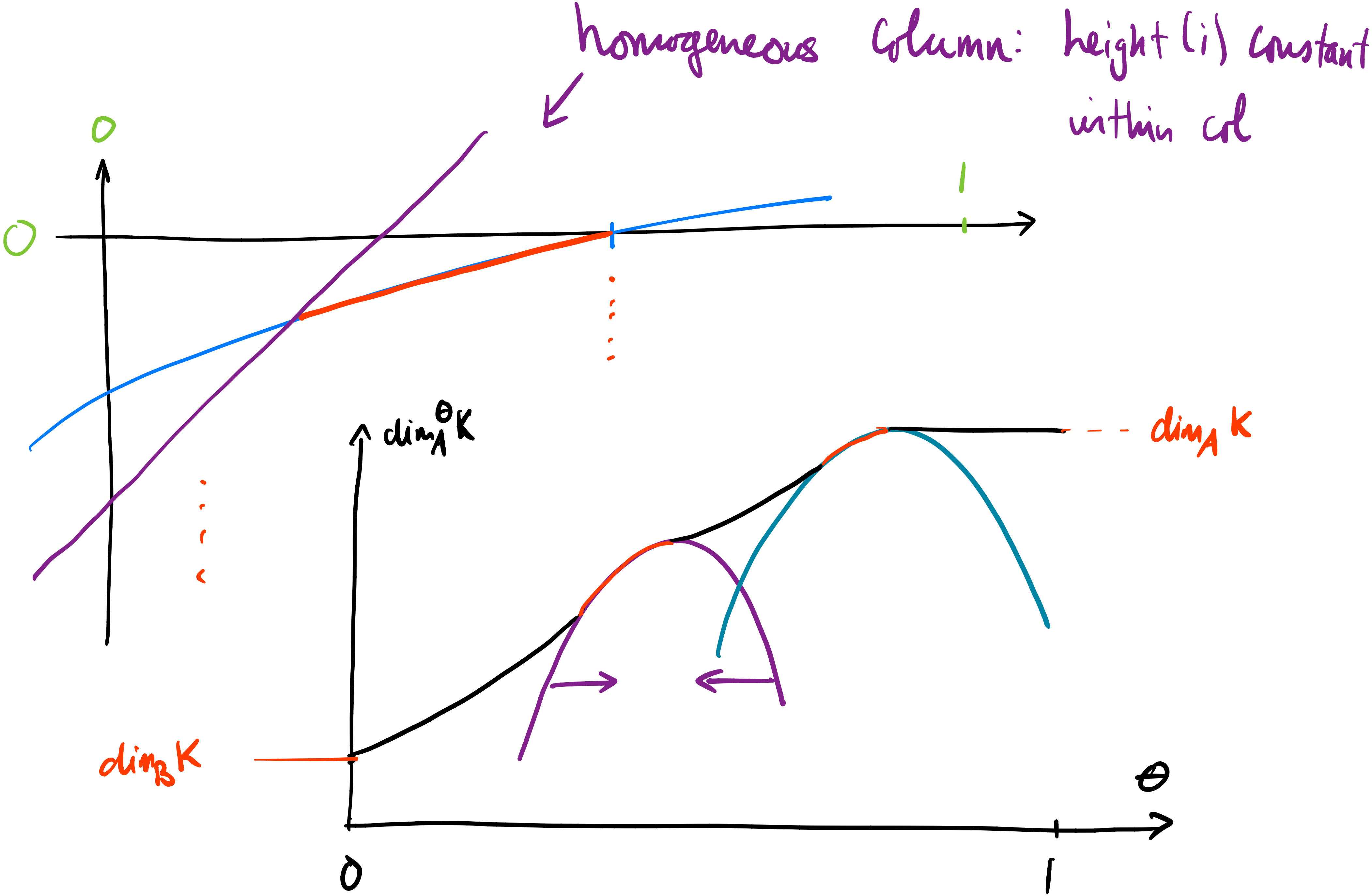
$$= \inf\{t \in \mathbb{R} : t\alpha - T(t)\}.$$

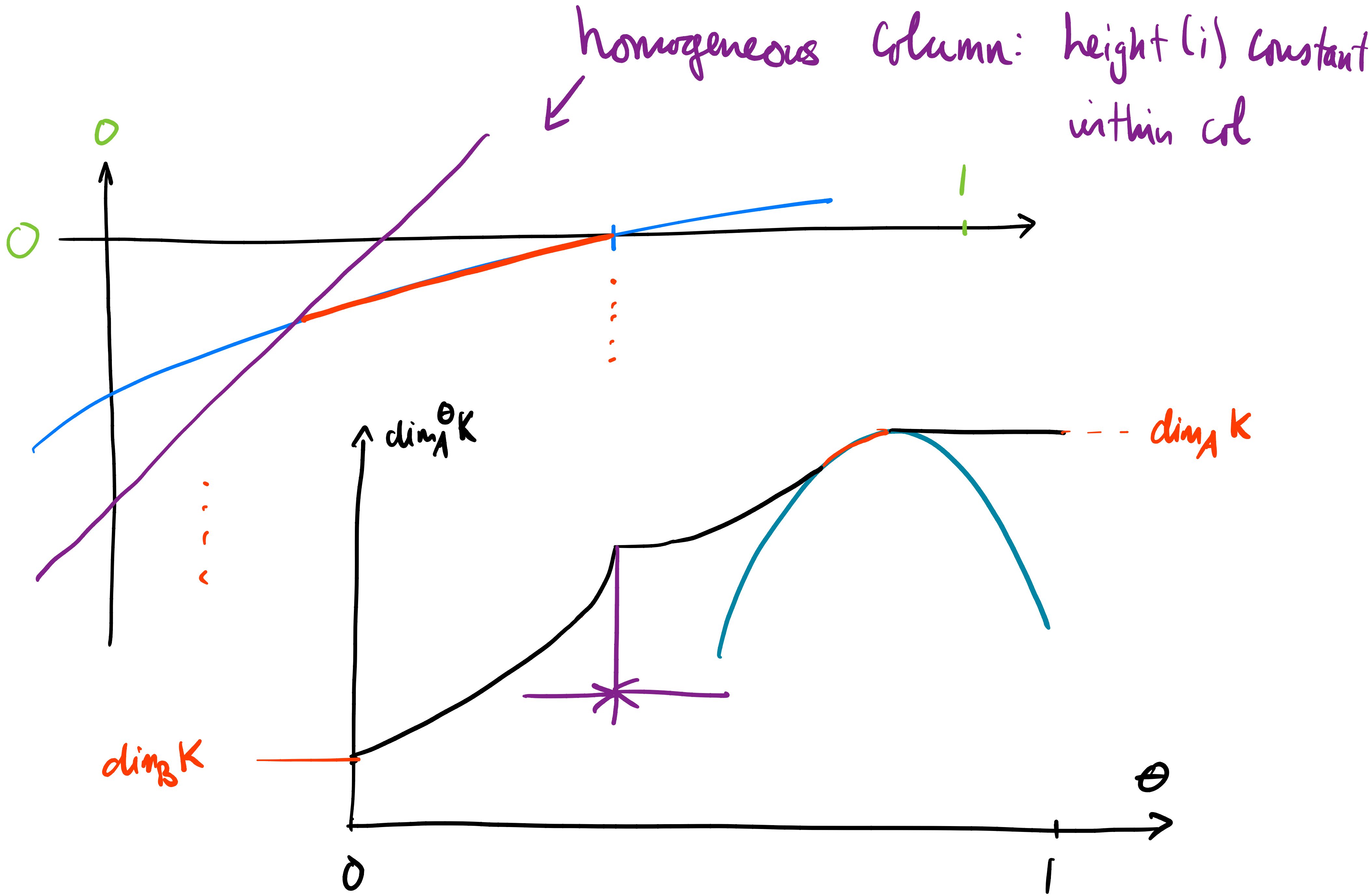
$$\phi(\theta) = \frac{\frac{1}{\theta} - 1}{1 - \frac{1}{\text{max log eccentricity}}}$$

"Smooth parameter change"

$$\dim_A^\theta K = \dim_B \operatorname{proj}(K) + \frac{\tau^*(\phi(\theta))}{\phi(\theta)}$$




homogeneous column: height(i) constant within col
0
0
1
dim$_A^\theta$ K
dim$_A$ K
dim$_B$ K
θ
0
1


homogeneous column: height(i) constant within col
0
0
1
$\dim_A^\theta K$
$\dim_A K$
$\dim_B K$
θ
0
1

Proof ideas

- large deviations / method of types: Subdivide covering argument into cases parametized by compact metric space
- obtain formula for $\dim_A^\theta K$ as non-smooth non-convex optimization
- solve optimization using "topological Lagrange multipliers"