Final project information

FRACTAL GEOMETRY AND DYNAMICS

Project expectations and evaluation.

- The final project is worth 50 points, with 25 points for the written component and 25 points for the oral component.
- Each topic likely contains too much material for a single project, so you should choose a subset of the material which you believe constitutes a cohesive whole (and is most interesting to you).
- The written component must be typeset and should occupy 5-10 pages with a reasonable layout (similar to this file, i.e. single-spaced, 12 point font, reasonable margin). Within reason, the report should contain all of the mathematical detail, though of course you should include statements without proof if the proof is very long, or tangential to the topic at hand.
- The oral presentation should take around 45 minutes (with a hard limit of 60 minutes). This should be approximately 30 minutes of content, and another 15 to 20 minutes for answering questions. You can present in any way that you prefer (board talk, slides, etc.).
- During the oral presentation, you should focus first and foremost on communicating the basic ideas of topic. It is most important to state the main theorems, to explain the context or significance of the result, and to give examples. Having done this, it is nice to give some ideas about the proof. It is important to keep in mind that 30 minutes is typically only enough time to summarize a topic, and nowhere near enough time to give detailed proofs.

Available topics. If you cannot find online PDFs for a topic that you are interested in, let me know and I will email them to you.

Dimensions of self-affine carpets. This topic concerns the dimension theory of probably the simplest example of an IFS attractor which is not a self-similar set. This is [BP17, Chapter 4] (original reference: [McM84]). You should at least prepare the derivation of the formulas for the Hausdorff and box dimensions.

Separation conditions for self-similar sets. This topic concerns separation conditions for self-similar sets. This is [BSS23, Chapter 4]. You should certainly discuss the open set condition and the strong open set condition. Also choose judiciously from:

- Weak separation condition
- Finite type condition
- Exponential separation condition

This topic is large enough that two people can work on it at the same time. If this happens, make sure to coordinate!

Inhomogeneous self-similar sets. These sets were introduced in Exercise Sheet 1, Question 2. The most important result is [Fra25, Theorem 2.1]. Afterwards, take a look at [Fra25, Theorem 4.2].

Assouad dimension and weak tangents. This concerns a notion of dimension that we have not studied in this course. See [Fra20, §2.1 and §5.1]. The most important results are [Fra20, Theorems 5.1.2 and 5.1.3].

Multifractal analysis for self-similar measures. We will talk quite a bit about local dimensions of measures in measures later in the course, but only in the typical sense. This topic looks at the structure of non-typical sets of local dimensions. This is [BSS23, Chapter 5]; the most important result is [BSS23, Theorem 5.1.1].

For a more topological viewpoint, an alternative option is to follow some notes that I wrote on this a few years ago. See [Rut23+, §3.1 and §3.4], in particular [Rut23+, Proposition 3.15].

Topological dynamical systems. This concerns a close relative to measure-preserving dynamical systems. See [Hoc17, Chapter 6]. The most important result is [Hoc17, Proposition 6.3.1]. [Hoc17, §6.5] is also important. It would be great if you could also discuss how topological dynamics relate to the existence of self-similar measures.

References

[BSS23] B. Bárány, K. Simon, and B. Solomyak. *Self-similar and self-affine sets and measures*. Vol. 276. Math. Surv. Monogr. Providence, RI: American Mathematical Society (AMS), 2023. zbl:1543.28001.

[BP17] C. J. Bishop and Y. Peres. *Fractals in probability and analysis*. Camb. Stud. Adv. Math. **162**. Cambridge: Cambridge University Press, 2017. zbl:1390.28012.

[Fra20] J. M. Fraser. *Assouad dimension and fractal geometry*. Camb. Tracts Math. **222**. Cambridge: Cambridge University Press, 2020. zbl:1467.28001.

[Fra25] J. M. Fraser. *Inhomogeneous attractors and box dimension*. In: vol. 825. Contemp. Math. Providence, RI: Amer. Math. Soc., 2025, 67–85. doi:10.1090/conm/825/16509.

[Hoc17] M. Hochman. *Notes on ergodic theory*. 2017.

[McM84] C. McMullen. *The Hausdorff dimension of general Sierpiński carpets*. Nagoya Math. J. **96** (1984), 1–9. zbl:0539.28003.

[Rut23+] A. Rutar. *Multifractal analysis via Lagrange duality*. Preprint. arxiv:2312.08974.